Unimechanics Discovering the Least Square Method Defects and Paradoxicalness

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In data processing, the least square method (LSM) [1] by Legendre and Gauss is practically the unique known one applicable to contradictory problems. But universal mathematics [2-5] has discovered that the LSM is based on the absolute error not invariant by equivalent transformations of a problem and loses any sense by possibly noncoinciding physical dimensions (units). The LSM simply mixes data without their adequately weighing, iterating, flexibility, and justification. The LSM paradoxically gives greater (even absolute) errors by smaller absolute values in approximation.

Universal mechanics [3] has additionally discovered further defects and paradoxicalness of the LSM. Consider its typical simplest approach. Minimizing the sum of the squared differences of the alone preselected coordinates (e.g., ordinates in a two-dimensional problem) of the graph of the desired approximation function and of every given data depends on this preselection, ignores the remaining coordinates, and provides no coordinate system rotation invariance and hence no objective sense of the result. Moreover, the LSM is correct by constant approximation or no data scatter only and gives systematic errors increasing together with data scatter and the deviation (namely declination) of an approximation from a constant.

Place the origin O of the coordinate system Oxy at the center of gravity of any planar data point set F mirror-symmetric with respect to Ox so that for the moments of inertia of F, $J_x = \int_F y^2 dF < J_y = \int_F x^2 dF$, $J_{xy} = \int_F xy dF = 0$. Fix initial Oxy as Ox'y' and rotate set F with Oxy about O by any angle α (Figures 1, 2). In Ox'y' with $x' = x \cos \alpha - y \sin \alpha$, $y' = x \sin \alpha + y \cos \alpha$, $c = \tan \alpha$, fit F via the LSM line $y' = kx' = (\tan \beta)x'$:

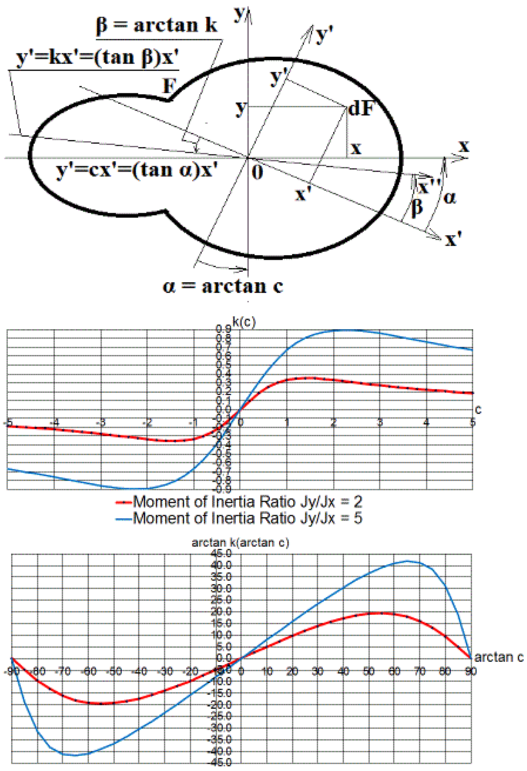

Figure 1. The LSM Rotation Bounds

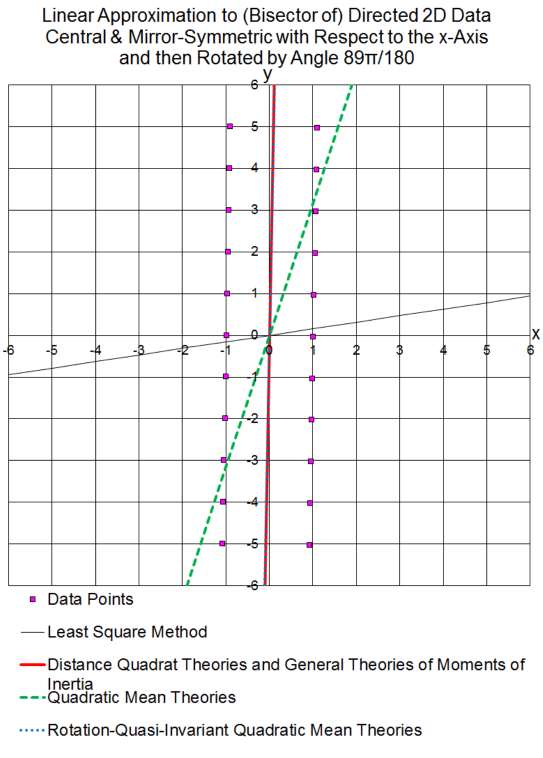

Figure 2. Initially F ={(0,±1), (±1,±1), … , (±5,±1)} in the initial system Oxy

$$ {}_{x'}^{2}S(k) = \int_F (kx' - y')^2 dF = \min;\ d_{x'}^{2}S(k)/dk = 0;\ \int_F 2x'(kx' - y')dF = 0;\ k = \int_F x'y'dF / \int_F x'^2 dF; $$

$$ \int_F x'y'dF = \sin \alpha \cos \alpha \int_F (x^2 - y^2)dF + (\cos^2 \alpha - \sin^2 \alpha) \int_F xydF = c(J_y - J_x)/(1 + c^2); $$

$$\int_F x'^2 dF = \cos^2\alpha \int_F x^2 dF - 2\sin\alpha\cos\alpha \int_F xy dF + \sin^2\alpha \int_F y^2 dF = (J_x c^2 + J_y)/(1 + c^2);$$
$$k = c(J_y - J_x)/(J_x c^2 + J_y),\ k_{max} = 0.5(J_y - J_x)/(J_x J_y)^{1/2} \text{ at } c_{max} = (J_y/J_x)^{1/2}.$$

Nota bene: By increasing α from 0 over arctan c_{max} to 90°, the LSM gives β increasing from 0 to arctan k_{max} and then suddenly decreasing to 0, respectively. The LSM slope k_{max} is about 0.34 and 0.89 whereas arctan k_{max} is about 19.47° and 41.79° only, respectively. Distance quadrat theories and general theories of moments of inertia give correct results even by great data scatter, e.g. in aeronautical fatigue.

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